

A calculation method for inert gas tension based on a multi-tissue model

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Abstract

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Introduction: This study developed a mathematical model capable of accurately estimating inert gas tension within human tissues under continuously changing ambient pressure.

Methods: Based on Haldane's decompression theory and a multi-tissue model, and grounded in the principle that the rate of gas diffusion is proportional to the pressure gradient, a differential equation was derived and solved. Analytical solutions were obtained for four prototypical ambient pressure patterns: constant pressure, exponential compression, linear compression, and linear decompression. The model's validity was confirmed by comparison with the piecewise approximation method using a sufficiently small step interval.

Results: The model allows precise calculation of theoretical tissue inert gas tension at any given time. In air-breathing fast buoyant ascents, for fast tissues with half-times of 30 s and 80 s, the inert gas tensions calculated using the conventional method (374 kPa and 233 kPa) were notably higher than those obtained via the proposed model (331 kPa and 217 kPa), with a maximum difference of 13%. The proposed model closely matched the piecewise method (1 s interval), differing by only 1 kPa. For tissue half-times exceeding 5 min, all three methods produced similar results. In a standard air diving decompression schedule, both the proposed and conventional methods yielded initial decompression stop depths rounded to 18 m, showing no significant difference.

Conclusions: The model demonstrates clear advantages for fast tissues, and under varying ambient pressures, establishes a theoretical foundation for the dynamic prediction of tissue inert gas tension across diverse diving scenarios. It can be utilised in safety assessments of fast buoyant ascent, offering substantial theoretical relevance and practical applicability.

Introduction

Inert gases, characterised by their low chemical reactivity and lack of involvement in metabolic processes, exist exclusively in a physically dissolved form within the body. Within the context of diving medicine, these gases encompass nitrogen, helium, neon, argon, krypton, and xenon. Nitrogen, constituting the predominant component of atmospheric air, represents the principal inert gas encountered in conventional air diving. The human body neither metabolises nor regulates nitrogen, thereby posing distinct challenges in the field of diving medicine. In the early twentieth century, Haldane and his colleagues conducted analyses of air diving experiences alongside animal studies to elucidate the behaviour of nitrogen within the body during diving. This work culminated in the formulation of decompression theory, which serves as the foundational framework for contemporary understanding of inert gas dynamics in diving medicine.

Under stable ambient pressure conditions, gas exchange within bodily tissues attains an equilibrium state. When ambient pressure changes, nitrogen diffuses bidirectionally

between the alveoli, bloodstream, and tissues, from blood to tissues during compression, and from tissues to alveoli during decompression. The rate of diffusion is directly proportional to the pressure gradient. The time required for an inert gas to reach 50% saturation in a tissue is defined as the half-saturation time (or half time), which varies according to tissue composition. Haldane's multi-tissue model categorises tissues into five hypothetical compartments with half-times of 5 min, 10 min, 20 min, 40 min, and 75 min, independent of pressure. These compartments are referred to as “*theoretical tissues*”.¹ The model remains a cornerstone in the global diving community, underpinning recreational, technical, commercial, and military diving, with modern refinements dating to the 1960s.^{2–4}

The functional definition of theoretical tissues is straightforward: their uptake and elimination of inert gases proceed in a fully symmetrical fashion. Studies have shown that the random movement and exchange of inert gas molecules between regions of different concentrations are driven by local pressure gradients, specifically, the difference between arterial inert gas tension and instantaneous tissue inert gas tension. To quantitatively describe the kinetics of

inert gases in the human body, a conventional mathematical model with respect to time t has been established, as shown in Eq. (1).¹⁻⁶

$$P_T = P_a + (P_{T_i} - P_a)e^{-kt} \quad (1)$$

where:

P_T is the inert gas tension in a given tissue at time t ;
 P_a is the arterial inert gas tension at time t (assumed to be equal to the inert gas tension in the breathing gas);
 P_{T_i} is the initial tissue inert gas tension;
 k is the saturation rate constant, related to the half-saturation time of the corresponding tissue.

The conventional model demonstrates accuracy solely under conditions of constant ambient pressure. However, actual diving scenarios involve continuous fluctuations in pressure during both descent (characterised by increasing pressure) and ascent (characterised by decreasing pressure). Existing methodologies face challenges in precisely capturing the dynamics of inert gas tension throughout these phases. In practice, several approximations are frequently employed, including adding descent time to bottom time to approximate total underwater exposure for tissue inert gas saturation; using the average of initial and final pressures during ascent as a surrogate for a constant equivalent pressure; or segmenting continuous pressure changes into multiple intervals of steady pressure. These approximations implicitly assume that the human body is subjected instantaneously to a constant-pressure environment, thereby disregarding the continuous nature of pressure variation. Given that instantaneous pressurisation or depressurisation is physiologically unfeasible, such simplifications inevitably lead to discrepancies between theoretical predictions and actual physiological responses. To address this limitation, the present study aimed to develop a concise mathematical model capable of accurately calculating inert gas tension in human tissues under conditions of continuously varying ambient pressure.

Methods

Let $f(x)$ denote the ambient pressure, specifically the pressure of the breathing gas, as a function of time x , and let $g(x)$ represent the pressure within a particular tissue over the same temporal variable, with $g'(x)$ signifying the rate of change of $g(x)$. According to the theoretical framework underpinning this analysis, variations in $f(x)$ elicit corresponding changes in $g(x)$, which serve to reduce the pressure difference between $g(x)$ and $f(x)$. Moreover, the rate of change $g'(x)$ is directly proportional to this pressure gradient. These considerations lead to the derivation of Eq. (2).

$$g'(x) = k[f(x) - g(x)] \quad (2)$$

Where:

The function $f(x)$ represents the ambient pressure, predicated on the assumption that arterial inert gas tension is equal to

the inert gas tension in the breathing gas. It is expressed in kPa. The corresponding inert gas tension is then obtained by multiplying $f(x)$ by the fractional concentration of the inert gas in the gas mixture. For clarity and ease of understanding, this study defines 100 kPa as one standard atmosphere and assumes that pressure increases by 100 kPa for every 10-metre descent in water depth. This assumption is made independently of the water's density.

The function $g(x)$ represents the pressure within a given tissue, measured in kPa. Its derivative, $g'(x)$, denotes the rate of change of tissue pressure with respect to time, expressed in $\text{kPa}\cdot\text{s}^{-1}$.

The parameter k is defined as the saturation rate constant, a dimensionless quantity with positive values.

From Eq. (2), Eq. (3) is derived:

$$g'(x) + kg(x) = kf(x) \quad (3)$$

Eq. (3) denotes a first-order nonhomogeneous linear differential equation, and its standard form is given by Eq. (4):

$$\frac{dy}{dx} + P(x)y = Q(x) \quad (4)$$

Its general solution is given by Eq. (5):

$$y = Ce^{-\int P(x)dx} + e^{-\int P(x)dx} \int Q(x)e^{\int P(x)dx} dx \quad (5)$$

By comparing Eq. (5) with Eq. (3), we have $P(x) = k$ and $Q(x) = kf(x)$. Substituting these into Eq. (5) yields:

$$g(x) = e^{-kx} \left(\int kf(x)e^{kx} dx + C \right) \quad (6)$$

Here, C is an arbitrary constant that is determined by the initial conditions. For any ambient pressure function $f(x)$, Eq. (6) represents the general solution of $g(x)$.

CONSTANT AMBIENT PRESSURE FUNCTION

Consider the function $f(x) = P_1$, where P_1 represents a constant ambient pressure to which an individual is exposed for a specified duration. Substituting $f(x) = P_1$ into Eq. (6) yields Eq. (7):

$$g(x) = P_1 + \frac{C}{e^{kx}} \quad (7)$$

When $x = 0$ s, substituting into Eq. (7) gives $g(0) = P_1 + C$, so $C = g(0) - P_1$. At this point, the saturation is 0%. Here, $g(0)$ represents the known initial tissue pressure.

As $x \rightarrow +\infty$, substituting into Eq. (7) yields $g(+\infty) = P_1$, indicating 100% saturation. Over the entire time course, $g(x)$ increases from $g(0)$ to P_1 .

Let T be the time at which 50% saturation is reached, corresponding to the half-saturation time of a given theoretical tissue. Substituting $C = g(0) - P_1$ into $g(T)$ yields $k = \ln 2/T$. Substituting these into Eq. (7) yields:

$$g(x) = P_1 - \frac{P_1 - g(0)}{2^{\frac{x}{T}}} \quad (8)$$

Eq. (8) is consistent with Eq. (1), and the commonly used saturation formula in diving medicine corresponds to these two equations, as shown in Eq. (9):

$$S = (1 - 0.5^n) \times 100\% \quad (9)$$

EXPONENTIAL AMBIENT PRESSURE FUNCTION

Consider the function $f(x) = P_0 \times 2^{x/n}$, which represents an individual exposed to an initial ambient pressure P_0 , where the pressure doubles every n seconds. Substituting $f(x) = P_0 \times 2^{x/n}$ into Eq. (6) yields:

$$g(x) = \frac{nkP_0}{\ln 2 + nk} \cdot 2^{\frac{x}{n}} + C \cdot e^{-kx} \quad (10)$$

Given that $g(0)$ is known, substituting $x = 0$ into Eq. (10) yields:

$$C = g(0) - \frac{nkP_0}{\ln 2 + nk} \quad (11)$$

Substituting Eq. (11) into Eq. (10) yields:

$$g(x) = \frac{nkP_0}{\ln 2 + nk} (2^{\frac{x}{n}} - e^{-kx}) + g(0)e^{-kx} \quad (12)$$

Generally, $g(0) = P_0$ — this indicates that, at the moment ambient pressure begins to change, tissue pressure is already in full equilibrium with the ambient pressure. If, at that point, the individual has experienced repeated diving or multiple compression/decompression cycles in a chamber, the value of $g(0)$ should be determined based on actual conditions. Substituting $g(0) = P_0$ into Eq. (11) yields:

$$C = \frac{P_0 \ln 2}{\ln 2 + nk} \quad (13)$$

Substituting Eq. (13) into Eq. (10) yields:

$$g(x) = P_0 \frac{nk \cdot 2^{\frac{x}{n}} + \ln 2 \cdot e^{-kx}}{\ln 2 + nk} \quad (14)$$

LINEAR AMBIENT PRESSURE FUNCTION

1. Linear ambient pressure decrease

Consider the function $f(x) = P_1 - ax$, which represents an individual initially exposed to an ambient pressure P_1 , with the pressure decreasing at a rate of a kPa per second, where $a > 0$. Substituting $f(x) = P_1 - ax$ into Eq. (6) yields:

$$g(x) = P_1 - \frac{a(kx - 1)}{k} + C \cdot e^{-kx} \quad (15)$$

Given that $g(0)$ is known, substituting $x = 0$ s into Eq. (15) yields:

$$C = g(0) - P_1 + \frac{a}{k} \quad (16)$$

Substituting Eq. (16) into Eq. (15) yields:

$$g(x) = P_1 - \frac{a(kx - 1)}{k} + \left(g(0) - P_1 + \frac{a}{k} \right) e^{-kx} \quad (17)$$

2. Linear ambient pressure increase

Consider the function $f(x) = P_0 + bx$, which represents an individual initially exposed to an ambient pressure P_0 , with the pressure increasing at a rate of b kPa per second, where $b > 0$. Substituting $f(x) = P_0 + bx$ into Eq. (6) yields:

$$g(x) = P_0 + \frac{b(kx - 1)}{k} + C \cdot e^{-kx} \quad (18)$$

Given that $g(0)$ is known, substituting $x = 0$ s into Eq. (18) yields:

$$C = g(0) - P_0 + \frac{b}{k} \quad (19)$$

Substituting Eq. (19) into Eq. (18) yields:

$$g(x) = P_0 + \frac{b(kx - 1)}{k} + \left(g(0) - P_0 + \frac{b}{k} \right) e^{-kx} \quad (20)$$

Generally, $g(0) = P_0$. In this case, $g(x) = P_0 + b(kx - 1)/k + b \times e^{-kx}/k$.

In fact, Howel, Weber and Vann et al. have already provided analytical solutions for tissue inert gas tension under linear compression and decompression conditions,⁷ and the results independently derived in this study are in complete agreement with their conclusions.

The definitive mathematical model is presented in Table 1. When the breathing gas is compressed air, it is necessary to multiply by 0.79 (or approximately 0.8) when calculating tissue nitrogen tension.

Table 1

Mathematical model based on different forms of ambient pressure variation; see text for additional symbol definitions

Forms of ambient pressure variation	Ambient pressure function	Tissue pressure function (multiply by 0.79 to obtain tissue nitrogen tension)	Remarks
Constant pressure	P_1	$g(x) = P_1 - \frac{P_1 - g(0)}{2^{\frac{x}{T}}}$	P_1 : initial ambient pressure, generally not atmospheric pressure T : half-saturation time of a given tissue $g(0)$: initial tissue pressure
Exponential compression	$P_0 \times 2^{x/n}$	$g(x) = \frac{nkP_0}{\ln 2 + nk} (2^{\frac{x}{n}} - e^{-kx}) + g(0)e^{-kx}$	n : pressure doubles every n seconds during the compression phase k : saturation rate constant P_0 : initial ambient pressure, generally atmospheric pressure $g(0)$: initial tissue pressure
Linear decompression	$P_1 - ax$	$g(x) = P_1 - \frac{a(kx-1)}{k} + \left(g(0) - P_1 + \frac{a}{k}\right)e^{-kx}$	k : saturation rate constant P_1 : initial ambient pressure, generally not atmospheric pressure $g(0)$: initial tissue pressure
Linear compression	$P_0 + bx$	$g(x) = P_0 + \frac{b(kx-1)}{k} + \left(g(0) - P_0 + \frac{b}{k}\right)e^{-kx}$	k : saturation rate constant P_0 : initial ambient pressure, generally atmospheric pressure $g(0)$: initial tissue pressure

CRITERION FOR MODEL EVALUATION

The piecewise approximation method, a common approach for calculating inert gas tension, converges to the exact solution as the step interval decreases. Therefore, to rigorously assess the accuracy of the proposed model, we compare its results with those obtained from the piecewise method using a step interval of 1 s. The primary metric for evaluating model performance is the level of agreement between the two methods, quantified as the maximum deviation in tissue nitrogen tension across the range of half-times considered.

Results**THEORETICAL TISSUE NITROGEN TENSIONS FROM THREE CALCULATION METHODS**

The initial pressure was 100 kPa, with exponential compression following $2^{1/4}$. The target depth was 100 m, followed by a high-pressure dwell time of 4 s. Decompression was performed at a rate of 30 kPa·s⁻¹, with

a nitrogen concentration of 80%. Six theoretical tissues, with half-saturation times ranging from 30 s to 20 min, were selected to calculate their surfacing nitrogen tension. During the ambient constant-pressure phase, all methods employed the classical half-saturation formula to determine nitrogen tension. For the ambient compression and decompression phases, the computational approaches differed as follows:

Conventional method: the duration of compression or decompression was converted into an equivalent exposure at a constant pressure equal to the average of the initial and final pressures for that phase.

Piecewise method: the ambient compression or decompression process was divided into 1-s intervals, each treated as a constant-pressure step, with the classical half-saturation formula applied to each step.

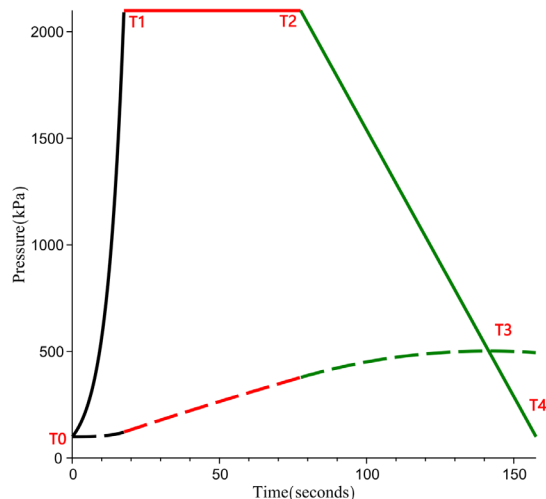
Proposed model method: parameters were directly substituted into the mathematical model developed in this study.

Table 2
Theoretical tissue nitrogen tension values obtained by different calculation methods

Calculation method	Theoretical tissue nitrogen tension (kPa)					
	30 s	80 s	120 s	5 min	10 min	20 min
Conventional method	374	233	189	127	104	92
Piecewise method	332	218	179	123	102	91
Proposed model method	331	217	178	123	102	91

Figure 1

Variation of ambient pressure and tissue pressure over time; solid black line – ambient pressure during the compression phase; solid red line – ambient pressure during the constant-pressure phase; solid green line – ambient pressure during the decompression phase; dashed black line – tissue pressure during the compression phase; dashed red line – tissue pressure during the constant-pressure phase; dashed green line – tissue pressure during the decompression phase



The calculation results are presented in Table 2.

NITROGEN TENSION VARIATION CURVES AND THEIR CHARACTERISTICS OBTAINED BY THE PROPOSED MODEL METHOD

The initial pressure was 100 kPa, with exponential compression following $2^{t/4}$. The target depth was 200 m, followed by a high-pressure dwell time of 60 s. Decompression was performed at a rate of 25 kPa·s⁻¹. The tissue half-time was 300 s, and the nitrogen concentration was 79%. Incorporating these parameters into the mathematical model produced the curves shown in Figure 1, with the corresponding results summarised in Table 3.

At time T_0 , the ambient compression phase begins. At this moment, ambient pressure equals tissue pressure, and the rate of increase of tissue pressure is zero.

The interval from T_0 to T_1 is the ambient compression phase. During this phase, tissue pressure increases in a concave manner, with both ambient pressure and tissue pressure rising rapidly. However, the rate of increase of tissue pressure is always lower than that of ambient pressure, causing the pressure difference between them to gradually widen.

At time T_1 , the pressure difference reaches its maximum, and simultaneously, the rate of increase of the tissue pressure function also peaks.

The interval from T_1 to T_2 is the ambient constant-pressure phase: ambient pressure remains constant, while tissue pressure increases in a convex manner (with T_1 as the inflection point), continuing to rise toward ambient pressure, but its rate of increase gradually slows.

The interval from T_2 to T_3 is the ambient decompression phase: ambient pressure decreases rapidly, while tissue pressure continues to rise, with the tissue pressure function remaining convex during this phase, until time T_3 .

At time T_3 , as shown in Figure 1, the ambient pressure curve intersects with the tissue pressure curve. At this point, the two pressures are equal, tissue pressure reaches its maximum, and its rate of increase drops to zero.

The interval from T_3 to T_4 is the ambient decompression phase: both ambient pressure and tissue pressure begin to decrease, but the rate of decrease of ambient pressure exceeds that of tissue pressure. Consequently, the pressure difference between them continues to widen, while the rate of decrease of tissue pressure gradually accelerates.

Discussion

The four types of ambient pressure functions selected in this study are all representative to some extent. When ambient pressure is constant, it indicates that the pressure remains stable over a specified period, corresponding to the constant-pressure phase. The associated tissue pressure function is consistent with the conventional saturation formula based on half-saturation time. When ambient pressure changes exponentially (typically with a base of two), it represents a rapid pressure increase, effectively

Table 3

Ambient pressure, tissue pressure, and tissue nitrogen tension at different time points; at time T0, ambient pressure begins to increase. At T1, the increase in ambient pressure stops and the constant-pressure phase begins. At T2, the constant-pressure phase ends and the ambient pressure decrease phase begins. At T3, tissue pressure and tissue nitrogen tension reach their maximum values. At T4, the ambient decompression phase ends

Category	Time point				
	0 s (T0)	17.57 s (T1)	77.57 s (T2)	141.48 s (T3)	157.57 s (T4)
Ambient pressure (kPa)	100.0	2100.0	2100.0	502.4	100.0
Tissue pressure (kPa)	100.0	122.4	378.4	502.4	495.0
Tissue nitrogen tension (kPa)	79.0	96.7	298.9	396.9	391.0

doubling every few seconds. This pattern is uncommon in standard diving conditions and primarily pertains to rapid compression in, for example, a submarine escape chamber. The linear function is further divided into two forms: constant-rate compression and constant-rate decompression. Constant-rate compression corresponds to a diver descending at a constant speed underwater or the pressure in a chamber increasing at a constant rate; constant-rate decompression corresponds to a diver ascending at a constant speed underwater or the pressure in a chamber decreasing at a constant rate. Collectively, these four functional forms effectively cover the typical pressure change patterns encountered in various diving scenarios.

AIR-BREATHING FAST BUOYANT ASCENT

The diving mode most suitable for the proposed model is the air-breathing fast buoyant ascent. It should be noted that fast buoyant ascent refers to a procedure in which personnel, after rapid compression in a chamber, directly enter seawater at a certain depth and ascend rapidly to the surface without any in-water decompression stops. During the ambient compression phase, personnel are in the chamber environment rather than the underwater environment; therefore, this is not a conventional diving mode. Hyperbaric exposure followed by fast buoyant ascent carries a risk of decompression sickness, with the initial bubble load considered to originate mainly from fast tissues.⁸ Decompression sickness (DCS) is believed to be caused by the formation of bubbles when tissue gases become supersaturated.⁹ Murphy et al. studied five novel pharmacokinetic structures with compartments coupled by perfusion and applied them to predict the probability and time of onset of decompression sickness in humans.¹⁰ Theoretically, the incidence of decompression sickness is closely related to tissue nitrogen tension; however, during fast buoyant ascent, the tension at and after surfacing, rather than during ascent, is considered the dominant factor. Since actual tissue nitrogen tension is difficult to measure, the calculation of theoretical tissue nitrogen tension is essential. The higher the accuracy of these calculations, the more accurate the assessment of personnel safety during fast buoyant ascent from different depths. The current commonly used calculation method estimates tissue nitrogen tension upon

surfacing based on the multi-tissue half-saturation theory. This method is uncontroversial for the constant-pressure phase. However, for the compression and decompression phases, where ambient pressure is not constant, the common practice is to convert these continuously changing phases into equivalent constant-pressure phases for approximate calculation. Because this treatment differs from the actual pressure variation pattern, discrepancies between calculated results and true physiological values are inevitable.

A fast buoyant ascent such as during submarine escape typically consists of three distinct phases: a compression phase, a constant-pressure phase, and a decompression phase. The compression and constant-pressure phases take place inside a chamber, where ambient pressure is fully controllable. Furthermore, the ascent rate during the decompression phase can be adjusted within certain limits through buoyancy control, thereby enabling accurate calculation of tissue nitrogen tension throughout the entire process. In practical calculations, the appropriate model is selected based on the specific characteristics of the ambient compression and decompression phases, and the final surfacing nitrogen tension is obtained through cumulative, segment-by-segment calculation.

Analysis of the data presented in Table 2 reveals several notable features when comparing the three calculation methods.

1) The difference between the conventional method and the other two approaches is most evident in fast tissues. For fast tissues with half-saturation times of 30 s and 80 s, the nitrogen tension values calculated by the conventional method are markedly higher than those obtained from the piecewise method and the proposed model. This indicates that during rapid ambient pressure changes, the conventional method overestimates the actual nitrogen tension in fast tissues because it neglects the continuity of ambient pressure variation.

2) The results obtained using the piecewise method and the proposed model are in close agreement. Across all six theoretical tissues, the values generated by these two methods are nearly identical, with a maximum difference

of only 1 kPa. This finding not only confirms the accuracy of the mathematical model developed in this study but also demonstrates that, in practical engineering applications or calculations, when the step interval is sufficiently small, the piecewise approximation method yields results sufficiently close to those of the exact model, offering practical utility.

3) When the half-saturation time reaches 5 min or more, the differences among the three methods are minimal, with a maximum difference not exceeding 1 kPa. This is because slow tissues are less sensitive to dynamic pressure changes, so the errors introduced by simplification are diluted.

In summary, the conventional method overestimates inert gas tension in fast tissues, rendering it unsuitable for approximate calculation in practical applications. The proposed method more accurately characterises the dynamics of tissue inert gas tension under continuous ambient pressure variations and exhibits strong concordance with the piecewise approximation method, thereby confirming its reliability and practical utility. It should be noted that in the piecewise method, reducing the time interval improves the accuracy of the results but increases computational complexity. When the step interval approaches zero, the outcomes converge with those obtained from the proposed model. Furthermore, as shown in Figure 1, the maximum nitrogen tension occurs at time T_3 rather than at T_4 (surfacing); during the interval from T_3 to T_4 , the tissue is already undergoing decompression, indicating that surfacing nitrogen tension is not the maximum. In contrast, the conventional method assumes that the tissue remains in a compression state throughout the entire process.

CONVENTIONAL DIVING

A conventional diving process typically consists of one or more descent (compression) phases, one or more bottom (constant-pressure) phases, and multiple ascent phases. During ascent, the diver's movement between decompression stops constitutes the decompression phase, while the stationary periods at these stops constitute the constant-pressure phase.

The proposed method enables accurate and convenient calculation of tissue inert gas tension during compression and decompression phases. A conventional air diving decompression schedule is used as an example for illustration: a diver descends from the surface to a depth of 50 m at a constant rate over 3 min, and then remains at that depth for 13 min of bottom work. The diver breathes compressed air, with a nitrogen percentage of 80%. A conventional air diving process generally consists of a descent phase, a bottom phase, multiple travel phases, and staged decompression phases. In the conventional method, the formula for the constant-pressure phase is directly applied to calculate tissue nitrogen tension during the descent and bottom phases, yielding nitrogen tensions of

216 kPa and 437 kPa for 5 min tissue at 3 min (end of descent) and 16 min (end of bottom phase), respectively. In contrast, using the linear compression mathematical model, the nitrogen tension of the same tissue at 3 min is 153 kPa, which is only 70.8% of the value obtained by the conventional method. During the subsequent constant-pressure phase (3 min to 16 min), the calculation of nitrogen tension is the same as that in the conventional method, giving a value of 426 kPa at 16 min, approximately 97.5% of the conventional method's result. The difference between the two methods at the end of the constant-pressure phase is small, mainly because the constant-pressure phase gradually reduces the deviation accumulated in the previous phase — the longer the constant-pressure phase, the closer the results of the two methods become.

Taking the safe coefficient of supersaturation as 1.6, the first decompression stop depth calculated by the conventional method is 17.2 m, while that obtained using the linear compression model is 16.6 m. After rounding, both methods give a first decompression stop depth of 18 m. It should be noted that during ascent, except for the first phase, the remaining ascent phases are often neglected in conventional calculations because of their short duration, making it impossible to optimise these phases using the proposed model. In summary, the principal advantage of the proposed model lies in its ability to accurately calculate tissue inert gas tension during ambient compression and decompression phases. In conventional diving applications, however, divers spend most of their time in the ambient constant-pressure phase, and the compression and decompression phases account for a relatively small proportion of the total time, resulting in no significant overall difference in the final decompression schedule. Moreover, the calculation principles of decompression schedules are inherently conservative, with depths and durations being rounded up by default when determining decompression stops, which further reduces the practical difference between the two methods.

Conclusions

The mathematical model, developed on the premise that the diffusion rate of a gas is proportional to the pressure gradient, provides an accurate and convenient calculation method that addresses the limitations in calculating inert gas tension during ambient compression and decompression phases. It offers a theoretical framework for the dynamic prediction of tissue inert gas tension across a range of diving scenarios. The exponential model, in particular, provides a significant improvement in the fidelity of the model to the likely physiological state. Compared with the conventional exponential response function employed during constant-pressure phases, the accuracy of this method improves with an increased proportion of ambient compression and decompression phases, as well as with shorter tissue half-saturation times.

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